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# **GRAVITY MODEL IMPROVEMENT INVESTIGATION**

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## 1. SCIENTIFIC/ TECHNOLOGICAL DESCRIPTION OF THE INVESTIGATION

The outstanding feature of the GEOS-C mission from the standpoint of Ocean Geoid Determination is the fact that it provides the prospect of conducting two independent kinds of experiments to determine the ocean geoid with a spatial resolution of about five degrees. These are the experiments based on the altimeter and the satellite-to-satellite tracking systems. Each of these approaches is, by itself, entirely new and hence provides an independent check of existing information. The primary GEOS-C Ocean Geoid Determination Investigation, therefore, is the combined investigation of the mean sea level geopotential surface by means of the altimeter and the satellite-to-satellite tracking system.

The altimeter data, when calibrated and corrected, e.g., for sea state and other effects, constitute measures of the distance between the GEOS-C spacecraft and the ocean surface. Knowledge of the satellite altitude relative to a reference ellipsoid and knowledge of the oceanographic departures of the sea surface topography from the geoid will then permit the determination of the geoid. The chief problem is expected to be the determination of the satellite orbital altitude. The primary tracking tools for doing this are the satellite-to-satellite tracking system and precision lasers. Data from these systems and the others tracking GEOS-C, including C-Band radars, USB range and range rate stations and Tranet Doppler stations, will be used to find the satellite height.

The satellite-to-satellite tracking system itself can sense the gravity field accelerations directly by observing the perturbing effects on the velocity and position of GEOS-C. The investigation based on tracking between GEOS-C and ATS-F is analogous to the one which has already been used to discover the lunar mascons by tracking lunar orbiters from earth. The spatial resolution of this approach is a function of the satellite altitude. It is estimated to be of the order of five degrees in the case of GEOS-C. The altimeter footprint diameter is of the order of a tenth of a degree, geocentric, hence its spatial resolution along the satellite track can, in principle, be of the order of a degree or less.

Thus the altimeter and satellite-to-satellite tracking approaches can, from the scientific standpoint, be used to conduct an ocean geoid determination investigation at the 5° spatial resolution level. Preliminary estimates indicate that both methods should be capable of height resolution of the order of five meters. The experiment using the two systems, the altimeter and the satellite-to-satellite tracking system, but aimed at the single objective of the determination of the ocean geoid, accordingly affords an exceptional opportunity for ocean geoid determination research.

This single scientific experiment will be proposed in terms of a combination of two investigations in order to conform to the investigation categories specified in the Proposal Briefing Information Handbook (ref. 1). These are an Ocean Geoid Determination Investigation, based primarily on altimeter sensing and orbital altitudes determined with the aid of satellite-to-satellite tracking data, and a Gravity Model Improvement Investigation based also on the use of satellite-to-satellite tracking data for sensing the gravity field directly.

The data requirements, theoretical formulation, computer program development, and preprocessing costs of the Ocean Geoid Determination Investigation should, with little or no additions, suffice for the companion Gravity Model Improvement Investigation. The actual analysis of the combination of altimeter and satellite-to-satellite tracking data and related information to determine the gravity field and geoid may involve a modest additional cost beyond that of an Ocean Geoid Determination Investigation of the type indicated in reference 1. A Gravity Model Improvement Investigation of this type is accordingly proposed here as the second part of the combined proposal.

The preceding discussion is included for convenience in both proposals and in order to indicate the fact that they constitute a combined set of investigation proposals for conducting what can be thought of as a single experiment. Each proposal will, as indicated, stress certain particular aspects of the overall combined experiment.

## 2. BACKGROUND

Satellite contributions to the determination of the current ocean geoid have spatial resolutions corresponding to half-wavelengths of about  $18^\circ$  (4, 7). Surface gravimetry achieves representations with finer resolution, in the  $1^\circ$  to  $5^\circ$  range, however, it covers only about half the ocean surface. Precisions of the order of five meters are obtained (4, 14, 19, 22). The GEOS-C altimeter and satellite-to-satellite tracking systems, which are expected to yield comparable results, will fill in the gaps and provide valuable independent knowledge where data now exist.

The value of satellite altimetry and satellite-to-satellite tracking in this connection has been pointed out by Kaula, Van Arx and their colleagues (19).

The ATS-F/GEOS-C satellite-to-satellite tracking experiment was first proposed in 1969 (ref. 23). The importance and value of this experiment to the overall GEOS-C mission has been reemphasized from time to time.

A related, ongoing program is the ATS-F/Nimbus-F tracking and Data Relay Experiment under Dr. F.O. Vonbun as the Principal Investigator. (39) This experiment is to be conducted at nearly the same time as the proposed combined Ocean Geoid Determination and Gravity Model Improvement Investigations which will make important use of the ATS-F/GEOS-C satellite-to-satellite tracking capability. The ATS-F/Nimbus-F experiment is aimed at the tracking and data relay aspects of the satellite-to-satellite link. The ability to determine orbits of typical NASA spacecraft such as Nimbus will be studied in the ATS/Nimbus experiment. This is of special interest since the Nimbus satellite is not passive, its orbit being perturbed by control jet action.

Data from the ATS-F/GEOS-C satellite-to-satellite tracking system, when taken together with other precise geodetic tracking data, will make it possible to evaluate the satellite-to-satellite technique at the geodetic precision level of accuracy and to make essentially unrestricted use of the geodetically validated satellite-to-satellite tracking data to achieve gravity model improvement. Data from the ATS-F/Nimbus-F experiment which is not perturbed by control jet actions and appears to be of appropriate accuracy will also be utilized although the bulk of the gravity field and geoid data is expected to come from the ATS-F/GEOS-C satellite-to-satellite tracking system. The ATS/Nimbus data will be used to check the determination of gravity anomalies in certain areas since its orbit will have a different inclination and hence, it will cross a given feature under different orbital circumstances. This approach has been used for example in lunar mascon analyses (Cf. refs. 24, 33).

The special value and importance of the GEOS-C satellite-to-satellite data, as indicated elsewhere, will derive from the fact that it can be correlated directly with the GEOS-C altimeter information.

Since the hardware for the ATS/GEOS and ATS/Nimbus satellite-to-satellite links will be equivalent in many respects, aspects of the analysis having to do with the tracking systems, per se, smoothing, formats, etc. in each experiment will benefit from the corresponding work done in the other experiment. The investigation teams have overlapping membership to insure maximum benefit to both efforts.

### 3. OBJECTIVES

The general objective of this investigation is to improve the gravity model and hence the ocean geoid. A specific objective of this portion of the combined investigation is the determination of the gravity field and geoid with a space resolution of approximately  $5^\circ$  and a height resolution of the order of five meters. An additional objective is to determine the geoid in selected regions with a space

resolution of about a degree and a height resolution of the order of a meter or two.

The short-term objectives include the study of the gravity field in the GEOS-C calibration area outlined by Goddard, Bermuda, Antigua, and Cape Kennedy, and also in the eastern Pacific area which is viewed by ATS-F. A second short-term objective is to evaluate the potential of satellite-to-satellite tracking data obtained when the high satellite is in different positions. As ATS-F drifts from  $94^{\circ}\text{W}$  to  $35^{\circ}\text{E}$ , it will be possible to study anomalies in the neighborhood of points such as those at  $30^{\circ}\text{W}$  by means of satellite-to-satellite data taken at a number of different angles. The value of such data sets for gravity model improvement will be studied in terms of gravity anomalies in the eastern Pacific, the Atlantic, and the western Indian Ocean.

A longer term objective associated with the second year of operation of GEOS-C is the improvement of the gravity model fine structure in the Indian Ocean area when ATS-F is at  $35^{\circ}\text{E}$ .

This investigation constitutes the first test of the high-low satellite type of GRAVSAT mission which is being considered in connection with the emerging Earth and Ocean Physics Applications Program (EOPAP). An in-orbit evaluation of the GRAVSAT-GEOPAUSE satellite-to-satellite tracking approach will be conducted as part of this investigation.

The experiment hardware has an accuracy of 0.08 cm/s for a ten-second integration interval, and 0.02 to 0.04 cm/sec for integration intervals in the 30-40 second range. The 40-second integration interval corresponds to about half of an orbital track over a  $5^{\circ}$  square. Thus, two independent measurements can be made in such squares using this integration interval. In principle, much longer time intervals, corresponding to the entire passage across a  $5^{\circ}$  square, might also be useful for gravity studies. The accuracy estimates will be based on the shorter smoothing intervals, which tend to give more conservative values.

A satellite at a 700 km altitude passing over an array of  $5^{\circ}$  squares has a sensitivity coefficient of about 0.08 cm/s corresponding to a ten milligal anomaly according to Schwarz (25, 40). He finds that the sensitivities vary roughly as the reciprocal of the altitude in this general range hence, for the GEOS-C orbit at an altitude of 843 km, the corresponding anomaly would be twelve milligals. The corresponding geoidal undulation obtained from Stokes' formula is of the order of five meters for a feature in the  $5^{\circ}$  to  $6^{\circ}$  range. (Cf., e.g., ref. 41.)

The chief orbital constraint is the one associated with the ground track spacing of about  $5^\circ$ , which corresponds to the spatial resolution capability of a satellite at this altitude. The present orbit was chosen so as to give ground track shifts of about  $5^\circ$  per day, hence the nominal orbit will satisfy this constraint. The corresponding constraint on data acquisition is the one requiring that a mesh or grid of nearly parallel tracks spaced  $5^\circ$  apart be laid down in both directions, i.e., south-north and north-south, over the regions to be studied, e.g., those cited above. It is clearly important to obtain altimeter data during these same passes since two purposes are served by this dual data taking. The satellite-to-satellite tracking data will improve the accuracy of the determination of the altimeter orbit altitude required for proper analysis of the altimeter experiment. Secondly, the simultaneous gathering of both altimeter and satellite-to-satellite tracking data will permit regions of interest to be observed and studied by two basically different tracking systems. This combined investigation will, as indicated above, make a fundamental contribution to the long-range EOPAP goals.

The chief justification for the use of the proposed satellite-to-satellite tracking and analysis techniques is the fact that they were used with striking success to discover the lunar mascons. (33) An estimate of the spatial resolution obtainable can be derived from the analysis of Schwarz, results of which are shown in Figure 1. (25). A spatial resolution capability of approximately five degrees is indicated.

#### 4. APPROACH

The concept of the investigation, as indicated above, is to utilize both GEOS-C altimeter and satellite-to-satellite tracking data to achieve gravity model improvement. The altimeter will provide data which are essentially geometric in character. The details of the investigative approach for this type of data are indicated in the companion Ocean Geoid Determination proposal. The satellite-to-satellite tracking data will be analyzed in at least two steps. Initially, the data traces from each pass will be studied directly to find indications and evidence for gravity anomalies. Rough correlation will be made with other information from other satellite and surface gravity sources. The approach is essentially the one already used in the analysis of the lunar mascons by the proposed Principal Investigator (Ref. 24). The second aspect of the approach will be to solve for parameters corresponding to mean values associated with  $5^\circ$  square regions. These parameters will be represented in terms of surface density distributions, sampling functions, or ensembles of mass points. Mass points appear to be perhaps somewhat better suited to the simultaneous modelling of surface geoid information generated by the altimeter and the orbital perturbations sensed by the satellite-to-satellite tracking system.

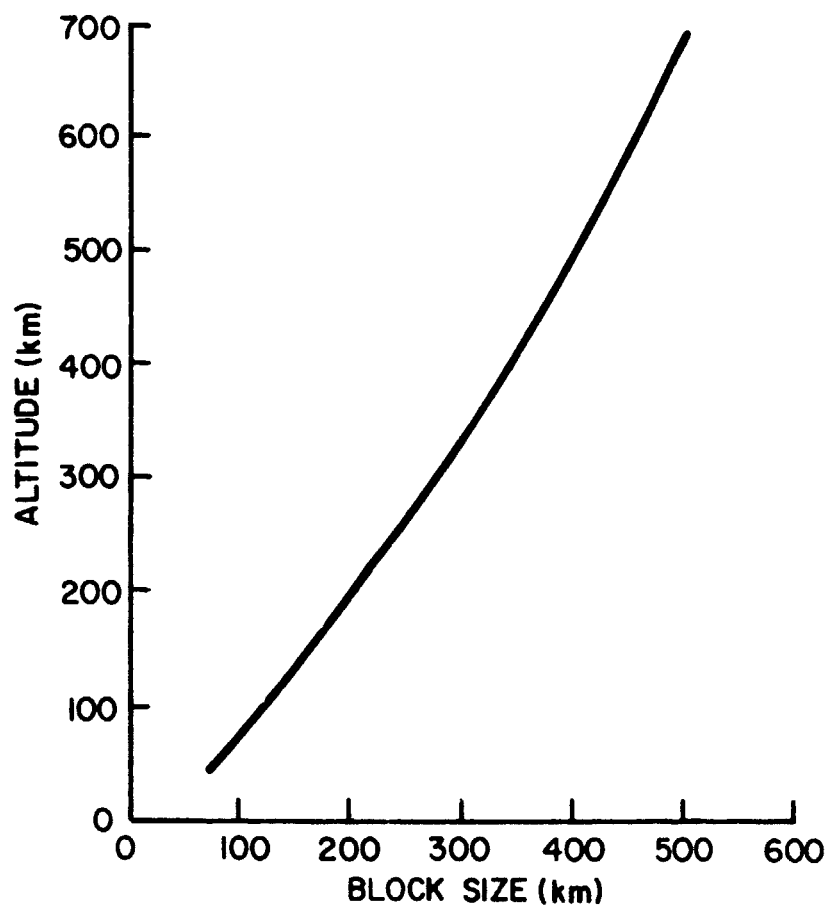


Figure 1

This point will, however, be looked into in more detail through the performance of analysis experiments. The sampling functions appear to be attractive since they facilitate relatively convenient transfer of information between representations associated with squares and the classical spherical harmonics. This approach will also be experimented with. The approach which appears most appropriate on the basis of these analyses will be used to specify the results. One or more of the alternative representations may also be used for this purpose too.

#### 5. DATA REQUIREMENTS

The data requirements have been discussed above, particularly in Section 4. The requirements are reviewed and summarized here. Geographical areas of interest are indicated in Table I in order of priority.



Table I

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| <p>A. GEOS-C Calibration Area - outlined by Goddard, Boston, Bermuda Antigua, Canal Zone, Cape Kennedy.</p> <p>B. The Remainder of the North Atlantic Ocean</p> <p>C. The Eastern Pacific Ocean Areas Viewed by ATS-F at 94°W.</p> <p>D. All Other Ocean Areas Viewed by GEOS-C</p> <p>E. The South Atlantic Ocean Areas Viewed by ATS-F While Drifting Between 94°W and 35°E.</p> <p>F. The Indian Ocean Areas Viewed by ATS-F at 35°E.</p> |
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Data should be taken in these regions in accordance with the following procedures.

Altimeter data passes should be scheduled so as to build up grids of tracks 5° apart in both the south-north and north-south directions in all the areas of Table I. The altimeter should take data at 1 second intervals.

Altimeter passes should be scheduled so as to build up, as feasible, grids of tracks 1° apart in both the south-north and north-south directions in the GEOS-C calibration area, i. e., region A of Table I.

Each altimeter track passing within 60° of the ATS-F subpoint should be tracked from ATS-F throughout a revolution pass and one immediately preceding or following it. Every SST pass should span the entire period of continuous visibility from ATS-F in a given revolution. The range rate integration interval should be 10 seconds. Range data should be taken at least once each 5 minutes at 1 second intervals for at least 20 seconds.

Each altimeter data pass should be accompanied by ground tracking, during a two revolution interval centered in the mid-time of the altimeter pass, from all laser and Tranet Doppler stations and as power permits, all C-band and USB stations whenever GEOS-C is above 5° elevation. Laser data should be taken at 1 second intervals or the shortest convenient interval, whichever is longer. Tranet data should be taken at the shortest convenient interval. C-band and USB data should be at intervals of about 6-10 seconds, or as power resources permit. Altimeter passes in the GEOS-C calibration area should be

tracked by both lasers and collocated C-band radars throughout each pass whenever laser visibility occurs in any part of the pass.

The tracking of the ATS spacecraft by its own independent tracking system should be done in conjunction with satellite-to-satellite tracking. These data are important since they will help in distinguishing between effects of ATS and GEOS motions on the satellite-to-satellite tracking data. The frequency contents of the ATS and GEOS orbital perturbations are, fortunately, quite different. Nevertheless, ATS tracking data are expected to be of good use in sorting out what is actually going on as the first attempts are made to extract information from satellite-to-satellite tracking data.

It is requested that all data formats to be used be specified as far in advance of the launch as is practicable.

## 6. DATA HANDLING PLAN

The data handling plan is outlined in Figure 2 which is consistent with the terminology and time schedules of appendix B of reference 1.

The deliverable data products will consist of geoid profiles and geoid maps. Geoid profiles will be based on the data from individual altimeter and satellite-to-satellite data tracks which are isolated from one another in terms of spacing and hence do not collectively map a 2-dimensional region. Geoid maps will be based on sets of altimeter and satellite-to-satellite data tracks which are nearly evenly spaced, e.g., at 5° intervals, and in sequences which are uninterrupted or nearly so. The "First Look" Data Analysis phase of ref. 1 is expected to yield geoid profiles. It is anticipated that geoid maps will begin to become available in a later phase on the basis of data obtained from the first half year of operation of GEOS-C.

In order to permit correlation with results and analysis of other types of data, the computer program system will be provided with the capability for solving for parameters associated with surface density distributions (42, 43), mass points, and the sampling functions proposed by Giacaglia and Lundquist.(35) The preprocessors will be equipped to handle all appropriate correlative and housekeeping data associated with the satellite, e.g., its attitude, the tracking systems, e.g., offsets, and the environment, e.g., meteorological quantities.(34) The analysis will be conducted with the aid of the GEODYN program which is being appropriately modified to conduct analyses using altimeter and satellite-to-satellite tracking data and to incorporate the features just mentioned. The GSFC IBM 360/95 and 360/75 computers will be used. The geoid profiles and maps will be furnished on magnetic tape in formats which, it is anticipated,

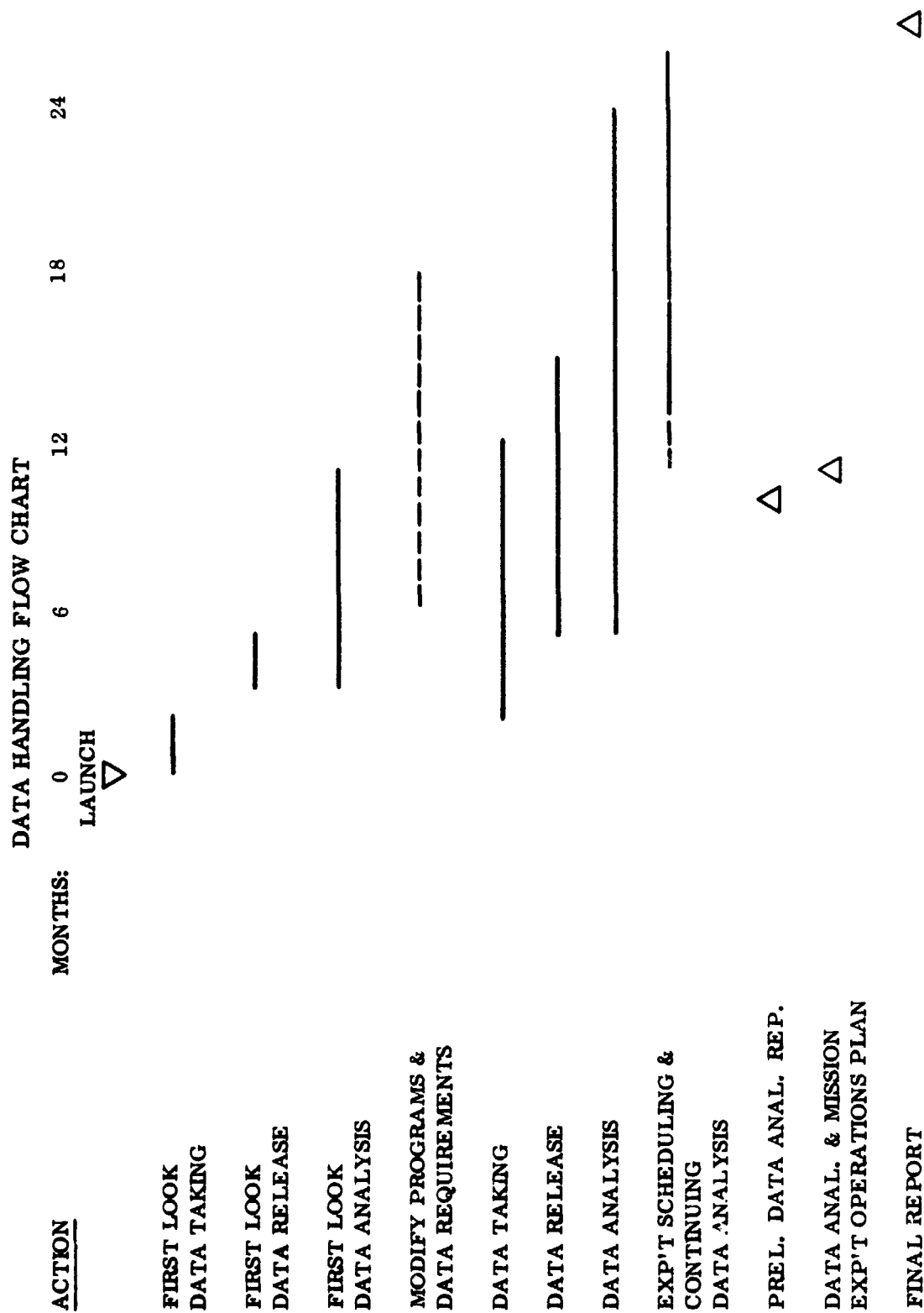


Figure 2

will specify the geoid height in meters as functions of geographic latitude and longitude. The profiles will be in the form of linear arrays, corresponding to the orbit tracks. The maps will be in the form of two-dimensional arrays.

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